Exercise 9

Find the closed form function for the following Taylor series:

$$f(x) = 2 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots$$

Solution

$$\begin{aligned}
f(x) &= 2 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots \\
f(x) &= 1 + 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots \\
f(x) &= 1 + \frac{(2x)^0}{0!} - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots \\
f(x) &= 1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(2x)^{2n}
\end{aligned}$$

Therefore,

$$f(x) = 1 + \cos 2x.$$